



الفصل الدراسي : الثاني

كود المقرر : EMP 002

الزمن : 75 دقيقة

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أسم المقرر : رياضيات (1-B)

أستاذ المادة: أ.د/ محمد سعد متولى

رقم المقعد:

الفصل:

اسم الطالب:

Evaluate $\int \operatorname{sech} x dx$

3 Ms

$$\begin{aligned} &= \int \frac{2}{e^x + e^{-x}} dx = 2 \int \frac{e^x}{1 + (e^x)^2} dx \\ &= 2 \operatorname{tanh}^{-1}(e^x) + C \end{aligned}$$

Evaluate $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

3 Ms

$$\begin{aligned} u &= \sin^{-1} x \quad \rightarrow \int e^u du = e^u + C \\ du &= \frac{dx}{\sqrt{1-x^2}} \\ &= e^{\sin^{-1} x} + C \end{aligned}$$

~~مذكرة مفبركة~~

~~محض ٢ اهار~~

Evaluate $\int \frac{dx}{\sqrt{(x^2-4)^3}}$

3 Ms

$$\text{Let } x = 2 \sec \theta$$

$$dx = 2 \sec \theta \tan \theta d\theta$$

$$I = \int \frac{2 \sec \theta \tan \theta}{4 \sqrt{4 \tan^2 \theta - 4}} d\theta = \frac{1}{4} \int \csc \theta \cot \theta d\theta$$

$$= -\frac{1}{4} \csc \theta + C$$

$$I = \frac{-x}{4 \sqrt{x^2-4}} + C$$

Evaluate $\int [\sinh^{-1} x]^2 dx$

3 Ms

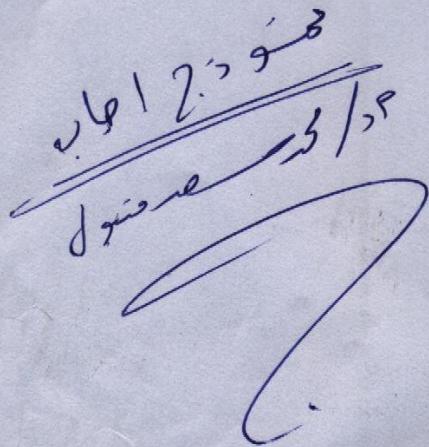
$$\begin{aligned} I &= x (\sinh^{-1} x)^2 - 2 \int \frac{x \sinh^{-1} x}{\sqrt{1+x^2}} dx \\ &\quad \left\{ \begin{array}{l} u = (\sinh^{-1} x)^2 \quad dv = dx \\ du = \frac{2 \sinh^{-1} x}{\sqrt{1+x^2}} dx \quad v = x \\ u = \sinh^{-1} x \quad dv = \frac{x}{\sqrt{1+x^2}} \\ du = \frac{dx}{\sqrt{1+x^2}} \quad v = \sqrt{1+x^2} \end{array} \right. \end{aligned}$$

$$I = x (\sinh^{-1} x)^2 - 2 \sqrt{1+x^2} \sinh^{-1} x + 2 \int dx$$

$$I = x (\sinh^{-1} x)^2 - 2 \sqrt{1+x^2} \sinh^{-1} x + 2x + C$$

Evaluate $\int_{-4}^4 \frac{\sinh x \tan^2 x}{\sqrt{1+x^4}} dx$ 3 Ms

$= 0$ because $\frac{\sinh x \tan^2 x}{\sqrt{1+x^4}}$ is odd function.



Evaluate $\int_2^9 \frac{\sqrt[3]{x-1}}{4+\sqrt[3]{(x-1)^2}} dx$ 3 Ms

$$x-1 = t^3 \Rightarrow dx = 3t^2 dt$$

$$\begin{cases} x=9 \\ x=2 \end{cases} \Rightarrow \begin{cases} t=2 \\ t=1 \end{cases}$$

$$\Rightarrow I = \int_{t=1}^2 \frac{3t^3}{t^3+4} dt = \int_1^2 \left(3t - \frac{12t}{t^3+4} \right) dt$$

$$= \left[\frac{3}{2}t^2 - 6 \ln(t^3+4) \right] \Big|_1^2$$

$$I = \frac{9}{2} - 6 \ln \frac{8}{5}$$

$$\boxed{\frac{t^2+4}{3t} \int \frac{3t^3}{3t^3+12t} dt}$$

Evaluate $\int \frac{\sin^2 x}{1-\tan x} dx$ 3 Ms

Let $\tan x = z$

$$dx = \frac{dz}{1+z^2}$$

$$\begin{matrix} \sqrt{1+z^2} \\ 1 \\ z \end{matrix}$$

$$I = \int \frac{\frac{z^2}{1+z^2}}{(1-z)} \cdot \frac{dz}{1+z^2} = \int \frac{-z^2}{(z-1)(z^2+1)^2} dz$$

$$= \int \left(\frac{-1/4}{z-1} + \frac{1/4z}{z^2+1} + \frac{1/4}{z^2+1} - \frac{1/2z}{(z^2+1)^2} - \frac{1/2}{(z^2+1)^2} \right) dz$$

$$= -\frac{1}{4} \ln z + \frac{1}{8} \ln(z^2+1) + \frac{1}{4} \operatorname{tan}^{-1} z + \text{C} \quad \text{and} \\ -\frac{1}{2} \int \sin x \cos x dx - \frac{1}{2} \int \cos^2 x dx$$

$$= \frac{1}{4} \ln(\csc x) + \frac{x}{4} - \frac{1}{4} \sin^2 x - \frac{x}{4} - \frac{1}{8} \sin 2x + C \\ \boxed{I = \frac{1}{4} \ln(\csc x) - \frac{1}{4} \sin^2 x - \frac{1}{8} \sin 2x + C}$$

$$\frac{-z^2}{(z-1)(z^2+1)^2} = \frac{A}{z-1} + \frac{Bz+C}{z^2+1} + \frac{Dz+E}{(z^2+1)^2}$$

$$z=1 \Rightarrow A=-\frac{1}{4}, \text{ and } z^2=0 \Rightarrow -\frac{1}{4}+B \Rightarrow B=\frac{1}{4} \\ \text{and } z^3=0 \Rightarrow -\frac{1}{4}+C \Rightarrow C=\frac{1}{4}, \text{ and } z^2=-1 \Rightarrow -\frac{1}{2}+D \\ z=0 \Rightarrow E=-\frac{1}{2} \Rightarrow D=-\frac{1}{2}$$

Find area bounded by

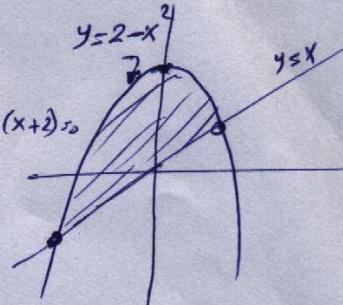
$$y = 2 - x^2 \text{ and } y = x$$

نطاق (area)

$$2-x^2 = x$$

$$\Rightarrow x^2+x-2 = (x-1)(x+2)$$

$$(1,1) \leq (2, -2)$$



$$\text{Area} = \int_{x=-2}^{x=1} [(2-x^2) - (x)] dx$$

$$= \left(2x - \frac{x^3}{3} - \frac{x^2}{2} \right) \Big|_{-2}^1$$

$$= \frac{9}{2} \text{ sq. units.}$$

\approx